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The impact of $\sigma(e^+e^- \to \text{hadrons})$ measurements at intermediate energies on the parameters of the Standard Model

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Abstract: We discuss the impact of precision measurements of $\sigma(e^+e^- \to \text{hadrons})$ in the center-of-mass range between 3 and 12 GeV, including improvements in the electronic widths of the narrow charmonium and bottonium resonances, on the determination of parameters of the Standard Model. In particular we discuss the impact of potential improvements on the extraction of the strong coupling constant α_s, on the evaluation of the hadronic contributions to the electromagnetic coupling $\alpha(M_Z)$, and the determination of the charm and bottom quark masses.

Keywords: Quark Masses and SM Parameters, Heavy Quarks Physics, QCD.

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1. Introduction

In view of the possibility for improved measurements of the total cross section in the energy region from approximately 3 GeV up to 12 GeV at CLEO [1] it seems useful to analyze the relevance of measurements at the different energy points for a variety of precision studies of the Standard Model. The issues discussed in this brief note are:

(i) the determination of the strong coupling α_s,

(ii) the contributions from this region to the running of the fine structure constant,

(iii) the determination of the charm and bottom quark masses.

We will not be concerned with the interpretation of the (narrow and wide) resonances in the context of quarkonium spectroscopy.

For definiteness we shall distinguish the following energy regions accessible by CLEO and the corresponding contributions to the parameters of interest:

(R1) The continuum below charm threshold $\langle 3\,\mathrm{GeV}, 2M_D\rangle$, excluding the narrow J/Ψ and Ψ' resonances,

(R2) the charm threshold region $\langle 2M_D, 5\,\mathrm{GeV}\rangle$ with its rapidly varying cross section and wide charmonium resonances,

(R3) the continuum region below the bottom threshold $\langle 5\,\mathrm{GeV}, 2M_B\rangle$, again excluding the narrow bottonium resonances $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$,

(R4) the bottom threshold region $\langle 2M_B, 11.5\,\mathrm{GeV}\rangle$ with its rapidly varying cross section and wide resonances,

energy region	(R1)	(R3)	(R5)
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	7.03 ± 0.07	41.72 ± 0.32	123.14 ± 0.24

Table 1: Contributions to $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ (in units of 10^{-4}) from the energy regions (R1), (R3) and (R5).

(R5) the continuum region starting at 11.5 GeV,

(R6) the electronic widths of the narrow resonances J/Ψ and Ψ',

(R7) the electronic widths of the narrow Υ resonances.

The separation points 5 GeV and 11.5 GeV should only be considered as approximate and are chosen such that pQCD is valid at and above these energies, an assumption to be tested by experiment.

2. α_s and the validity of perturbative QCD

Predictions for $R(s) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma_{\rm pt}$ based on pQCD are valid down to fairly low energies. At present the agreement between theory and experiment has been tested at the level of $2-4\%$ in the energy region between 3 and 10.5 GeV. This has led to a determination of α_s which already demonstrates the running of α_s as extracted from the same observables, albeit at vastly different energies. The results as derived from present experiments (including those derived from τ and Z decays) are displayed in figure 1. Measurements with precisions of 1%, would be nearly competitive with the determination of α_s from τ decays [2] ($\alpha_s(m_\tau) = 0.334 \pm 0.022$) and the hadronic Z-decay rate [3] ($\alpha_s(M_Z) = 0.1183 \pm 0.0027$) and would lead to a beautiful confirmation of its running from M_Z down to m_τ as is demonstrated in figure 1.

3. The continuum region and its relevance for the electromagnetic coupling

Detailed predictions based on pQCD are available for the continuum regions (R1), (R3) and (R5) (see [5]–[7] and references cited therein). Given α_s, the remaining uncertainty from unknown higher orders has been estimated to be around 2.5%, 1.5% and 2.5% for 3 GeV, 5 GeV and 11.5 GeV, respectively. The validity of pQCD at these points is taken for granted in all sum rule calculations (see below Section 5) and in the recent analyses of $\alpha(M_Z)$ [6, 8, 9] whereas the earlier papers employed pQCD only above 40 GeV (see, e.g. [10]).

The absolute contributions to $\Delta\alpha_{\rm had}^{(5)}$ from the regions (R1), (R3) and (R5) (up to 40 GeV) based on pQCD are listed in table 1. This has to be compared with a total contribution $\Delta\alpha_{\rm had}^{(5)} = 277.5 \pm 1.7 \times 10^{-4}$ [8]. However, as emphasized above, this is based on the (though well founded) *assumption* that pQCD is valid in this range and should be contrasted with the present experimental uncertainties of 4.3% ((R1), BES [11]), 4%

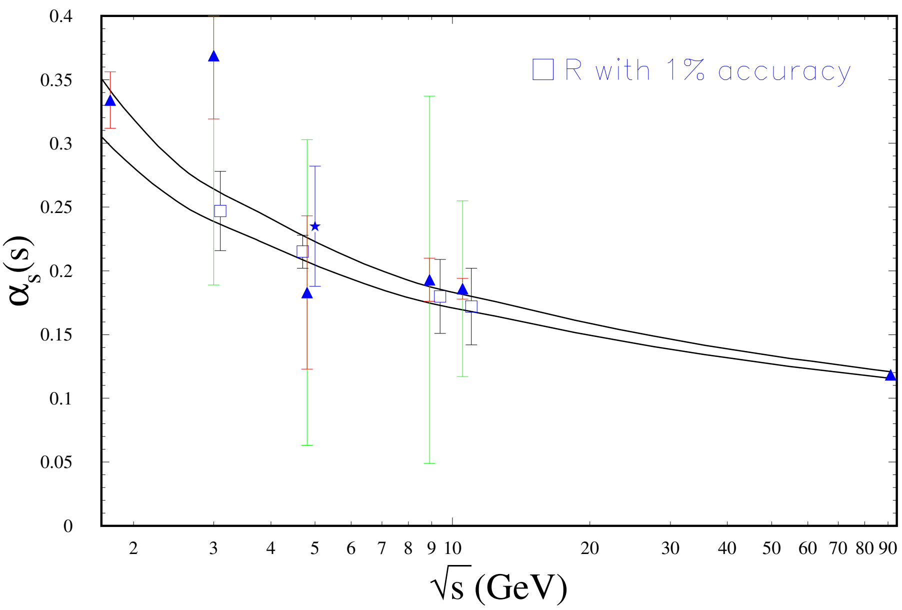

Figure 1: α_s as a function of $\sqrt{s}$. Results from τ [2] and Z decays [3] and those extracted in [4] from the R-ratio at 3 GeV, 4.8 GeV, 8.9 GeV and 10.52 GeV are shown. The two error bars on the data points indicate the statistical (inner) and systematical (outer) uncertainty. For the combined result (indicated by a star) at $\sqrt{s} = 5.0$ GeV only the error after adding the statistical and systematical uncertainty in quadrature is shown. For illustration $\delta\alpha_s$ reduced to the error as expected from $\delta R/R = 1\%$ are also shown (slightly displaced in order to make the presentation more visible).

((R3),MD1 [12]) and 2% (10.52 GeV, CLEO [13]). A measurement of $\sigma(e^+e^- \to \text{hadrons})$ at a few well chosen energy points would confirm or disprove the validity of pQCD in these regions and would allow to completely replace the theory driven analysis by precise experiments combined with interpolations based on pQCD or give additional support to the theory driven evaluations based on pQCD.

4. The impact of narrow resonances and the threshold region on $\Delta\alpha^{(5)}_{\rm had}$

Narrow resonances contribute to $\Delta\alpha^{(5)}_{\rm had}$ through

$$\Delta\alpha^{(5)}_R(M_Z^2) = \frac{3}{\alpha}\left(\frac{\alpha}{\alpha(M_R^2)}\right)^2 \frac{M_Z^2}{M_R^2}\frac{M_R\Gamma_{ee}}{M_Z^2 - M_R^2}\,. \tag{4.1}$$

The contribution from the Υ resonances is smaller than the one of charmonium resonances by approximately one order of magnitude, a consequence of the smaller charge of bottom quarks, and their larger mass.

The contribution from the lowest three charmonium resonances to $\Delta\alpha^{(5)}_{\rm had}$ and its present error is sizeable, $9.24 \pm 0.74 \times 10^{-4}$ to be compared with $56.90 \pm 1.10 \times 10^{-4}$ from the low energy region up to 1.8 GeV, and also in comparison to the total error of 1.68×10^{-4} [8].

The same holds true for the threshold region (R2). For $\Delta\alpha^{(5)}_{\rm had}$ a significant improvement has already been achieved by the BES collaboration, with their systematic error of roughly 4%. Nevertheless it would be desirable to reduce this error by another factor two. This would again allow to replace the theory driven treatment of the data as described in [8] by a purely experiment-based evaluation. The contribution from region (R4) is less important in this context, if one assumes the validity of pQCD for u, d, s and c production.

Hence the reduction of the systematic errors in the electronic widths of the charmonium resonances and in the charm threshold cross section to approximately 2% would lead to a significant reduction of the uncertainty in $\Delta\alpha_{\rm had}^{(5)}$. In this context it would not be necessary to arrive at this precision for every individual scan point: it is only the weighted integral which matters.

5. The determination of charm and bottom quark masses through moments

Let us, in the first part, concentrate on the determination of the charm quark mass.

The approach used in [4] to compute the charm (and bottom) quark mass is based on the use of low-order moments. This has the advantage that non-perturbative effects from the gluon condensate can be neglected and no resummation of the Coulomb singularities are required. As a consequence of the latter one can directly determine the $\overline{\rm MS}$ quark mass which is a big advantage as compared to those methods which intrinsically have to deal with the pole mass.

On the theoretical side the computation of the moments is reduced to the evaluation of the charm-quark contribution to the photon polarization function for which the first eight terms of the Taylor expansion are known analytically up to the three-loop order [14].

The quark mass can be extracted from the moments

$$\mathcal{M}_n^{\rm th} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2}\right)^n \bar{C}_n \,, \tag{5.1}$$

with coefficients $\bar{C}_n$ which depend logarithmically on m_c. On the experimental side the moments can be spit into three parts:

$$\begin{aligned} \mathcal{M}_n^{\rm exp} &= \int \frac{{\rm d}s}{s^{n+1}} R_c(s) \\ &= \mathcal{M}_n^{\rm exp,res} + \mathcal{M}_n^{\rm exp,cc} + \mathcal{M}_n^{\rm cont} \,, \end{aligned} \tag{5.2}$$

the contribution from the resonances J/Ψ and Ψ', the contribution from the charm threshold region ($3.73\,{\rm GeV} \le \sqrt{s} \le 4.8\,{\rm GeV}$), and the contribution from the continuum above $\sqrt{s} = 4.8\,{\rm GeV}$. For $\mathcal{M}_n^{\rm exp,cc}$ the BES-data [11] have been used. Due to the use of low-order moments there is still a sizeable contribution from $\mathcal{M}_n^{\rm cont}$. To be precise, it amounts to 31% (10%) for $n = 1$ ($n = 2$). A detailed decomposition of the individual contributions to the moments and the error can be found in table 2.

Currently there is no reliable data for $R(s)$ in the energy region above $\sqrt{s} = 4.8\,{\rm GeV}$. Thus, in [4] for this part the theoretical prediction for $R(s)$ has been used. This is motivated by the fact that there is very good agreement with experiment in those energy

	J/Ψ, Ψ'	charm threshold region	continuum	sum
n	$\mathcal{M}_n^{\mathrm{exp,res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp,cc}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)	0.2065(84)
2	0.1096(79)	0.0174(8)	0.0142(3)	0.1412(80)

Table 2: Experimental moments separated according to the contributions from the resonances, the charm threshold region and the continuum region above $\sqrt{s} = 4.8\,\mathrm{GeV}$.

regions where data is available (see [4]). Note, that the full mass dependence for $R(s)$ is known up to order α_s^2, and the first three expansion terms in m^2/s are available at order α_s^3. In [4] the relative error of $\mathcal{M}_n^{\mathrm{cont}}$ turned out to be 1.5% (2%) for $n = 1$ ($n = 2$). However, as emphasized before, this consideration is based of the validity of pQCD above 4.8 GeV. It would be of considerable importance to verify this assumption through a precise measurement.

The charm quark mass cited in [4] (obtained from $n = 1$) reads $m_c(3\,\mathrm{GeV}) = 1.027(30)$ which corresponds to $m_c(m_c) = 1.304(27)\,\mathrm{GeV}$. Almost 28 MeV of the uncertainty in $m_c(3\,\mathrm{GeV})$ is of experimental origin, i.e., comes from the error in $\mathcal{M}_n^{\mathrm{exp}}$. In case the uncertainty in $\mathcal{M}_n^{\mathrm{cont}}$ is increased to 10% this increases to 35 MeV. Conversely, assuming that the error on the electronic widths of the narrow resonances and the continuum could be reduced to 2%, and adding the two contributions with uncorrelated errors, the moments would be known with a relative precision of roughly 1.5%. This would lead to a final error on $m_c(m_c)$ of 10 to 15 MeV.

To summarize: a reliable measurement of $R(s)$ above the charm threshold region would be very important to cross check or even replace the use of the theoretical prediction for the evaluation of $\mathcal{M}_n^{\mathrm{cont}}$. This would require a scan of $R(s)$ for $4.8\,\mathrm{GeV} \leq \sqrt{s} \leq 7.5\,\mathrm{GeV}$. Since the cross section is flat in this region a measurement, e.g., at three or five different center-of-mass energies should be sufficient. Furthermore, the uncertainties in $\Gamma_{ee}(J/\Psi)$ and $\Gamma_{ee}(\Psi')$ and in the charm threshold region should be reduced to 2%. At the same time the separation of the u, d, s-background and the charm contribution would be highly desirable. This would allow to test the (plausible) assumption on the behaviour of the continuum cross section and would furthermore lead to a reduction of the error in m_c down to $10 - 15\,\mathrm{MeV}$.

Similar considerations apply to the determination of m_b. In this case the continuum (with the separation point presently chosen at 11.2 GeV [4]) plays an even more important role (see table 3 where the relative contribution to the lowest three moments from the three narrow resonances, from the threshold contribution up to 11.2 GeV and from the continuum above 11.2 GeV are listed). Thus the measurement of $\sigma(e^+e^- \to b\bar{b})$ at 11.4 GeV, where a large data sample has already been collected would be extremely useful for a test of pQCD at this energy point and thus for a reliable evaluation of the continuum contribution to the low moments. Given a 2% measurement of Γ_{ee} for the narrow resonances, a 2% measurement of the threshold region and a 2% measurement at 11.4 GeV, a determination of $m_b(m_b)$ to 30 MeV seems well feasible.

n	$\mathcal{M}_n^{\mathrm{exp,res}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp,thr}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(2n+1)}$
1	1.237(63)	0.306(86)	2.913(21)	4.456(121)
2	1.312(65)	0.261(72)	1.182(12)	2.756(113)
3	1.399(68)	0.223(61)	0.634(8)	2.256(108)

Table 3: Moments for the bottom quark system: $\mathcal{M}_n^{\mathrm{exp,res}}$ includes the contribution from $\Upsilon(1S) - \Upsilon(3S)$; $\mathcal{M}_n^{\mathrm{exp,thr}}$ includes the remaining threshold contributions up to 11.2 GeV; $\mathcal{M}_n^{\mathrm{cont}}$ represents the continuum above 11.2 GeV.

6. Summary

The importance of the improved determination of the cross section for charm and bottom production in their respective threshold region has been discussed. Both regions are of importance for measurements of α_s in the intermediate range. The charm region is of particular relevance for the hadronic contribution to the electromagnetic coupling at M_Z. An improved determination of m_c and m_b with a precision below 15 MeV and 30 MeV, respectively, seems within reach, once these cross sections are known to better than 2%. In general a precise determination of $\sigma(e^+e^- \to \text{hadrons})$ at a few selected points, e.g. 3 GeV, 3.73 GeV, 5 GeV, 10.5 GeV and 11.5 GeV combined with a scan through the threshold regions for charm and bottom production (and an evaluation of the weighted integrals) would be sufficient for this purpose. The additional measurement of $\sigma(e^+e^- \to \text{hadrons})$ at two or three selected points between 5 and 10 GeV could provide additional confidence in pQCD motivated interpolations.

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